REPORT No. 623

A STUDY OF THE TORQUE EQUILIBRIUM OF AN AUTOGIRO ROTOR

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SUMMARY

Two improvements have been made in the method developed in N. A. C. A. Reports Nos. 487 and 591 for the estimation of the inflow velocity required to overcome a given decelerating torque in an autogiro rotor. At low tip-speed ratios, where the assumptions necessary for the analytical integrations of the earlier papers are valid, the expressions therein derived are greatly simplified by combining and eliminating terms with a view to minimizing the numerical computations required. At high tip-speed ratios, by means of charts based on graphical integrations, errors inherent in the assumptions associated with the analytical method are largely eliminated.

The suggested method of estimating the inflow velocity presupposes a knowledge of the decelerating torque acting on the rotor; all available full-scale experimental information on this subject is therefore included.

INTRODUCTION

The results reported in reference 1 showed the agreement between the calculated and the experimental values of the forces and moments on an autogiro rotor to be unsatisfactory at tip-speed ratios greater than 0.3. An attempt to isolate the sources of the disagreement was made by comparing values of the analytically integrated expressions for thrust and accelerating torque from reference 1 with values obtained by graphical integration of the elemental forces. This comparison indicated that the simplifications introduced in reference 1 to make possible the analytical integration of the basic expression for accelerating torque are not valid at high tip-speed ratios. Consequently, the relation therein given between accelerating torque and inflow velocity results in an erroneous estimation of the inflow velocity required to overcome a given decelerating torque at high tip-speed ratios. Because the value of the inflow velocity so estimated must be used in calculating all the rotor forces and moments from the other expressions of reference 1, this error introduces corresponding errors into the predicted values for all the various items constituting the rotor performance.

In the present paper an attempt is made to establish a more satisfactory method of expressing the relation between the accelerating torque and the inflow velocity, so that the inflow velocity required to overcome a given decelerating torque at high tip-speed ratios may be readily estimated with an accuracy comparable with that previously attainable only at low tip-speed ratios. At the same time, the original method of estimating the inflow velocity at low tip-speed ratios is simplified by regrouping the terms in the torque equation in a manner designed to eliminate most of the tedious numerical computations previously required.

The method given presupposes a knowledge of the decelerating torque arising from the drag of the blade elements. Consequently all available full-scale experimental information on this subject is included as a guide to the designer until such time as a satisfactory method is devised for predicting decelerating torque from blade airfoil characteristics.

ANALYSIS

GENERAL

The values of the expressions for the forces and moments on an autogiro rotor, as derived in references 1 and 2, are all critically dependent on the mean value of the inflow velocity through the rotor disk. Any attempt to predict the performance of a particular rotor therefore requires, first, an accurate determination of the inflow velocity necessary to maintain steady autorotation of the rotor under the given conditions. If use is made of the physical requirement that the net torque on the rotor must be zero in steady autorotation, the estimation of the inflow velocity may be accomplished in two steps: first, the estimation of the decelerating torque on the rotor due to the profile drag of the blade elements; and, second, the estimation of the inflow velocity required to generate in the rotor an accelerating torque just equal to the estimated decelerating torque. The present paper being primarily concerned with the prediction of the inflow velocity when the decelerating torque is known, the second of these steps will first be considered.

Throughout the following analysis it will be assumed, as in reference 1, that the inflow velocity is uniform over the disk area and that the contribution of the radial-velocity components to the elemental forces is negligible. The notation used is identical with that of reference 2; for convenience, however, a list of symbols and their definitions has been included.

SYMBOLS AND DEFINITIONS

Ω, rotor angular velocity, radians per second.
R, blade radius.
V, forward speed.
α, rotor angle of attack, radians.
$\lambda\Omega R$, speed of axial flow through rotor.
$\mu\Omega R$, component of forward speed in plane of disk.
b, number of blades.
ψ, blade azimuth angle measured from down wind in direction of rotation, radians.
c, blade chord.
B, tip-loss factor.
xR, radius of blade element.
$u_T\Omega R$, velocity component at blade element perpendicular to blade span and parallel to rotor disk.
$u_P\Omega R$, velocity component at blade element perpendicular both to blade span and to $u_T\Omega R$.
$\phi=\tan^{-1}\dfrac{u_P}{u_T}$.
a, slope of lift coefficient against angle of attack of blade airfoil section (radian measure).
Q, rotor torque.
Q_a, accelerating rotor torque.
Q_d, decelerating rotor torque.
$C_Q=\dfrac{Q}{\rho\Omega^2\pi R^5}$
T, rotor thrust.
$C_T=\dfrac{T}{\rho\Omega^2\pi R^4}$
σ, solidity, ratio of total blade area to swept-disk area, $bc/\pi R$.
θ_0, blade pitch angle at hub, radians.
θ_1, difference between hub and tip pitch angles, radians.
θ_e, equivalent constant pitch angle, radians.
a_n, coefficient of $\cos n\psi$ in Fourier series expressing blade flapping angle, radians.
b_n, coefficient of $\sin n\psi$ in Fourier series expressing blade flapping angle, radians.
ϵ_n, coefficient of $\cos n\psi$ in Fourier series expressing periodic blade twist angle, radians.
η_n, coefficient of $\sin n\psi$ in Fourier series expressing periodic blade twist angle, radians.
I_1, mass moment of inertia of rotor blade about horizontal hinge.
$\gamma=\dfrac{c\rho aR^4}{I_1}$, mass constant of rotor blade.
M_W, weight moment of blade about horizontal hinge.

ACCELERATING TORQUE

The basic expression for the accelerating torque on an autogiro rotor is

$$Q_a=\frac{b}{2\pi}\frac{1}{2}\rho c\Omega^2R^4\sum_0^{2\pi}\Delta\psi\sum_0^{B}u_T^2C_L\frac{\sin\phi}{\cos^2\phi}x\Delta x \tag{1}$$

By definition

$$Q_a=C_{Q_a}\rho\Omega^2\pi R^5$$

and

$$\sigma=\frac{bc}{\pi R}$$

Then

$$\frac{C_{Q_a}}{\sigma}=\frac{1}{2}\left(\frac{1}{2\pi}\sum_0^{2\pi}\Delta\psi\sum_0^{B}u_T^2C_L\frac{\sin\phi}{\cos^2\phi}x\Delta x\right) \tag{2}$$

The indicated summation may be performed analytically on the assumptions that C_L varies linearly with angle of attack of the element, regardless of the magnitude of this angle, and that the angle ϕ is always so small that the sine, tangent, and angle are interchangeable. This integration has been covered in reference 1 for blades having a constant twist and has been extended in reference 2 to cover the more general case in which the blade twist varies periodically with the azimuth position of the blade. The final expression is:

$$\begin{aligned}\frac{C_{Q_a}}{\sigma}=\frac{a}{2}\Big\{&\lambda^2\left(\frac{1}{2}B^2-\frac{1}{4}\mu^2\right)+\lambda\left(\frac{1}{3}\theta_0B^3+\frac{2}{9\pi}\mu^3\theta_0+\frac{1}{4}\theta_1B^4\right.\\&\left.+\frac{1}{32}\mu^4\theta_1\right)+\mu\lambda a_1\left(\frac{1}{2}B^2-\frac{3}{8}\mu^2\right)+a_0^2\left(\frac{1}{4}\mu^2B^2-\frac{1}{16}\mu^4\right)\\&-\frac{1}{3}\mu a_0b_1B^3+a_1^2\left(\frac{1}{8}B^4+\frac{3}{16}\mu^2B^2\right)+b_1^2\left(\frac{1}{8}B^4+\frac{1}{16}\mu^2B^2\right)\\&-a_2\left(\frac{1}{4}\mu^2a_0B^2+\frac{1}{6}\mu b_1B^3\right)+\frac{1}{2}a_2^2B^4\\&+b_2\left(\frac{1}{8}\mu^2\theta_0B^2+\frac{1}{12}\mu^2\theta_1B^3+\frac{1}{6}\mu a_1B^3\right)+\frac{1}{2}b_2^2B^4\\&+\epsilon_0\left(\frac{1}{4}\lambda B^4+\frac{1}{12}\mu^2b_2B^3+\frac{1}{32}\mu^4\lambda\right)\\&+\frac{1}{2}\epsilon_1\left(-\frac{1}{4}\mu a_0B^4+b_1\left[\frac{1}{5}B^5+\frac{1}{12}\mu^2B^3\right]-\frac{1}{8}\mu a_2B^4\right)\\&+\frac{1}{2}\eta_1\left(\frac{1}{3}\mu\lambda B^3-a_1\left[\frac{1}{5}B^5-\frac{1}{12}\mu^2B^3\right]-\frac{1}{8}\mu b_2B^4\right)\\&+\frac{1}{2}\epsilon_2\left(\frac{1}{4}\mu a_1B^4+\frac{2}{5}b_2B^5\right)\\&+\frac{1}{2}\eta_2\left(-\frac{1}{6}\mu^2a_0B^3+\frac{1}{4}\mu b_1B^4-\frac{2}{5}a_2B^5\right)\Big\}\end{aligned} \tag{3}$$

When this expression is compared with equation (35) of reference 2, it must be remembered that the term $\dfrac{\delta}{4a}\left(1+\mu^2-\dfrac{1}{8}\mu^4\right)$ in equation (35) of reference 2 is in reality $2C_{Q_d}/\sigma a$. This quantity will equal $2C_{Q_a}/\sigma a$ in steady autorotation. Consequently, C_{Q_a}/σ will be $a/2$ times the sum of the remaining terms in equation (35) of reference 2.

Expressions for the Fourier coefficients of blade flapping, also derived in reference 2 (equations (22), (23), (24), (27), and (28)), are:

$$\begin{aligned}a_0=\frac{1}{2}\gamma\Big\{&\frac{1}{3}\lambda B^3+0.080\mu^3\lambda+\frac{1}{4}\theta_0\left(B^4+\mu^2B^2-\frac{1}{8}\mu^4\right)\\&+\frac{1}{5}(\theta_1+\epsilon_0)\left(B^5+\frac{5}{6}\mu^2B^3\right)+\frac{1}{8}\mu^2b_2B^2+\frac{1}{4}\mu\eta_1B^4\\&-\frac{1}{12}\mu^2\epsilon_2B^3\Big\}-\frac{M_W}{I_1\Omega^2}\end{aligned} \tag{4}$$

$$a_1=\frac{2\mu}{B^4-\frac{1}{2}\mu^2B^2}\left\{\lambda\left(B^2-\frac{1}{4}\mu^2\right)+\theta_0\left(\frac{4}{3}B^3+0.106\mu^3\right)\right.$$
$$+(\theta_1+\epsilon_0)B^4-\frac{1}{3}b_2B^3+\frac{\eta_1}{\mu}\left(\frac{2}{5}B^5+\frac{1}{2}\mu^2B^3\right)$$
$$\left.-\frac{1}{2}\epsilon_2B^4\right\} \tag{5}$$

$$b_1=\frac{4\mu}{B^4+\frac{1}{2}\mu^2B^2}\left\{a_0\left(\frac{1}{3}B^3+0.035\mu^3\right)+\frac{1}{6}a_2B^3\right.$$
$$\left.-\frac{\epsilon_1}{\mu}\left(\frac{1}{5}B^5+\frac{1}{12}\mu^2B^3\right)-\frac{1}{4}\eta_2B^4\right\} \tag{6}$$

$$a_2=\frac{\mu^2\gamma}{\gamma^2B^8+144}\left\{\lambda B\left(16+\frac{7}{108}\gamma^2B^8\right)+\theta_0B^2\left(\frac{46}{3}+\frac{7}{144}\gamma^2B^8\right)\right.$$
$$+(\theta_1+\epsilon_0)B^3\left(12+\frac{7}{180}\gamma^2B^8\right)-\frac{1}{30}\frac{\epsilon_1}{\mu}\gamma B^8+\frac{2}{5}\frac{\eta_1}{\mu}B^4$$
$$\left.+\frac{24}{5}\frac{\epsilon_2}{\mu^2}B^5+\frac{2}{5}\frac{\eta_2}{\mu^2}\gamma B^9\right\} \tag{7}$$

$$b_2=\frac{-\mu^2\gamma^2}{\gamma^2B^8+144}\left\{\frac{5}{9}\lambda B^5+\frac{25}{36}\theta_0B^6+\frac{8}{15}(\theta_1+\epsilon_0)B^7+\frac{2}{5\gamma}\frac{\epsilon_1}{\mu}B\right.$$
$$\left.+\frac{1}{30}\frac{\eta_1}{\mu}B^8+\frac{2}{5}\frac{\epsilon_2}{\mu^2}B^9-\frac{24}{5\gamma}\frac{\eta_2}{\mu^2}B^5\right\} \tag{8}$$

Substituting from equations (4), (5), (6), (7), and (8) into equation (3), retaining only terms of the order of μ^4 or lower, and regrouping

$$\frac{C_{Q_a}}{\sigma}=\frac{a}{2}\left\{K_1\lambda^2+\left[K_2\theta_0+K_3(\theta_1+\epsilon_0)+K_7\eta_1+K_{10}\epsilon_1+K_{11}\epsilon_2\right.\right.$$
$$\left.+K_{12}\eta_2+K_{24}\frac{M_W}{I_1\Omega^2}\right]\lambda+K_4\theta_0^2+K_5\theta_0(\theta_1+\epsilon_0)$$
$$+K_6(\theta_1+\epsilon_0)^2+K_8\theta_0\eta_1+K_9(\theta_1+\epsilon_0)\eta_1+K_{13}\theta_0\epsilon_1$$
$$+K_{14}\theta_0\epsilon_2+K_{15}\theta_0\eta_2+K_{16}(\theta_1+\epsilon_0)\epsilon_1+K_{17}(\theta_1+\epsilon_0)\epsilon_2$$
$$+K_{18}(\theta_1+\epsilon_0)\eta_2+K_{19}\epsilon_1^2+K_{20}\epsilon_1\eta_1+K_{21}\eta_1^2$$
$$+K_{22}(\epsilon_1\epsilon_2+\eta_1\eta_2)+K_{23}(\epsilon_1\eta_2-\eta_1\epsilon_2)+\frac{M_W}{I_1\Omega^2}\left[K_{25}\theta_0\right.$$
$$+K_{26}(\theta_1+\epsilon_0)+K_{27}\epsilon_1+K_{28}\eta_1+K_{29}\epsilon_2+K_{30}\eta_2$$
$$\left.\left.+K_{31}\frac{M_W}{I_1\Omega^2}\right]\right\} \tag{9}$$

where the coefficients K_1 to K_{31}, for which complete expressions are given in the appendix, are functions of μ, B, and γ only.

The magnitudes of the coefficients K_{24} to K_{31} are such that, when a value less than 0.01 is assigned to $M_W/I_1\Omega^2$, the resulting contribution of the terms involving these coefficients to C_{Q_a}/σ is negligible over the entire range of tip-speed ratios. The influence of γ on C_{Q_a}/σ is also of a secondary nature; comparison of values of the coefficients K_1 to K_{23} for $\gamma=0$ with those for $\gamma=15$ indicates that no appreciable error will arise from the use of a value of $\gamma=15$ for any conventional rotor at any attainable altitude. Inasmuch as the chord-span ratio of the blades used in present-day rotors varies only within small limits, it is possible to assign arbitrarily a value of 0.970 to the tip-loss factor B with the assurance that only minor departures from the recommended value of $1-\frac{1}{2}\frac{c}{R}$ (reference 1) will result.

By the use of the foregoing values for γ and B with extreme values of tip-speed ratio, pitch, twist, and inflow velocity, it can be shown that the terms involving K_{10}, K_{13}, K_{16}, K_{19}, K_{20}, K_{22}, and K_{23} are negligible. Of the remaining coefficients, K_{11}, K_{12}, K_{14}, K_{15}, K_{17}, K_{18}, and K_{21} are merely constants multiplied by μ^2; whereas, K_1 to K_9 involve two or more powers of μ.

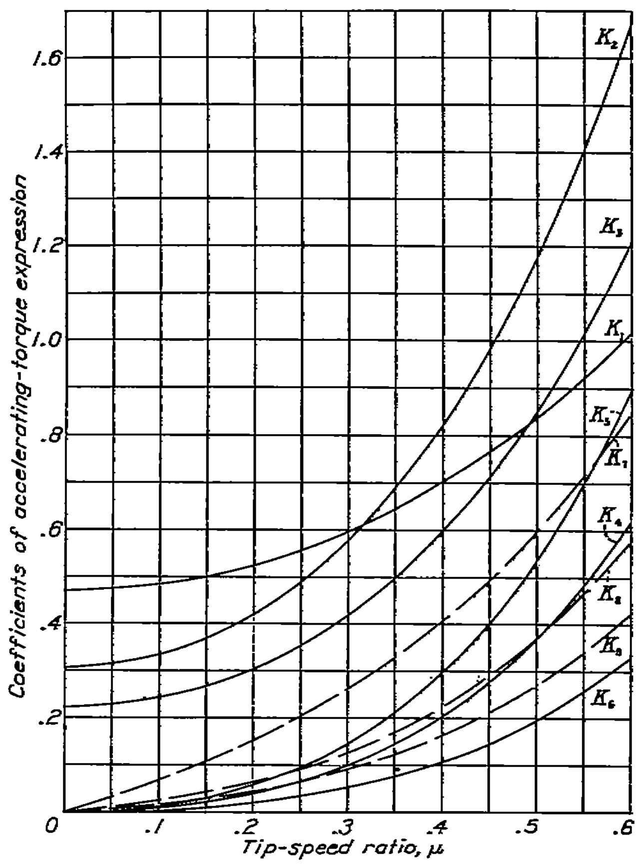
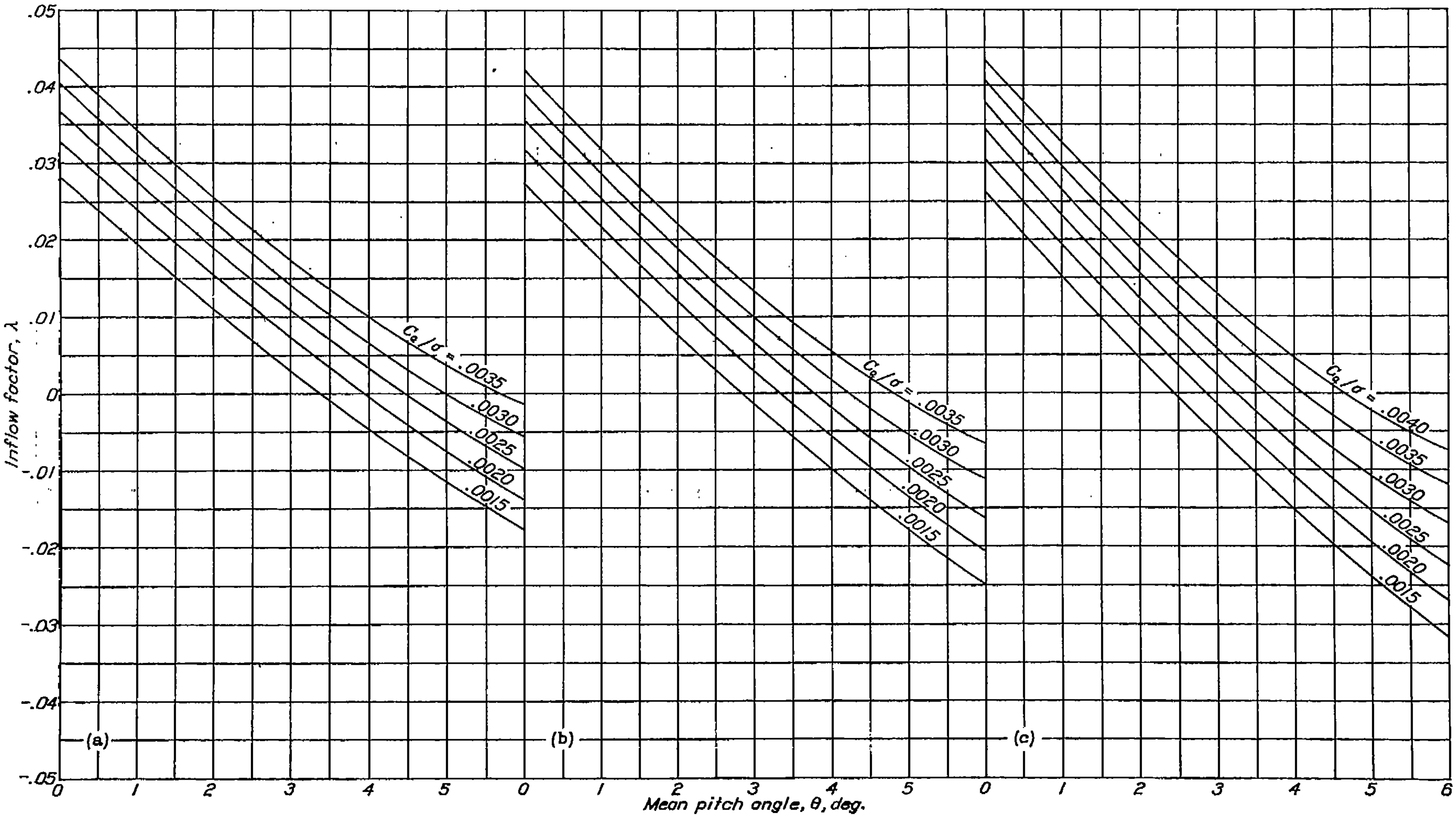


FIGURE 1.—Coefficients K_1 to K_9 of accelerating-torque expression.

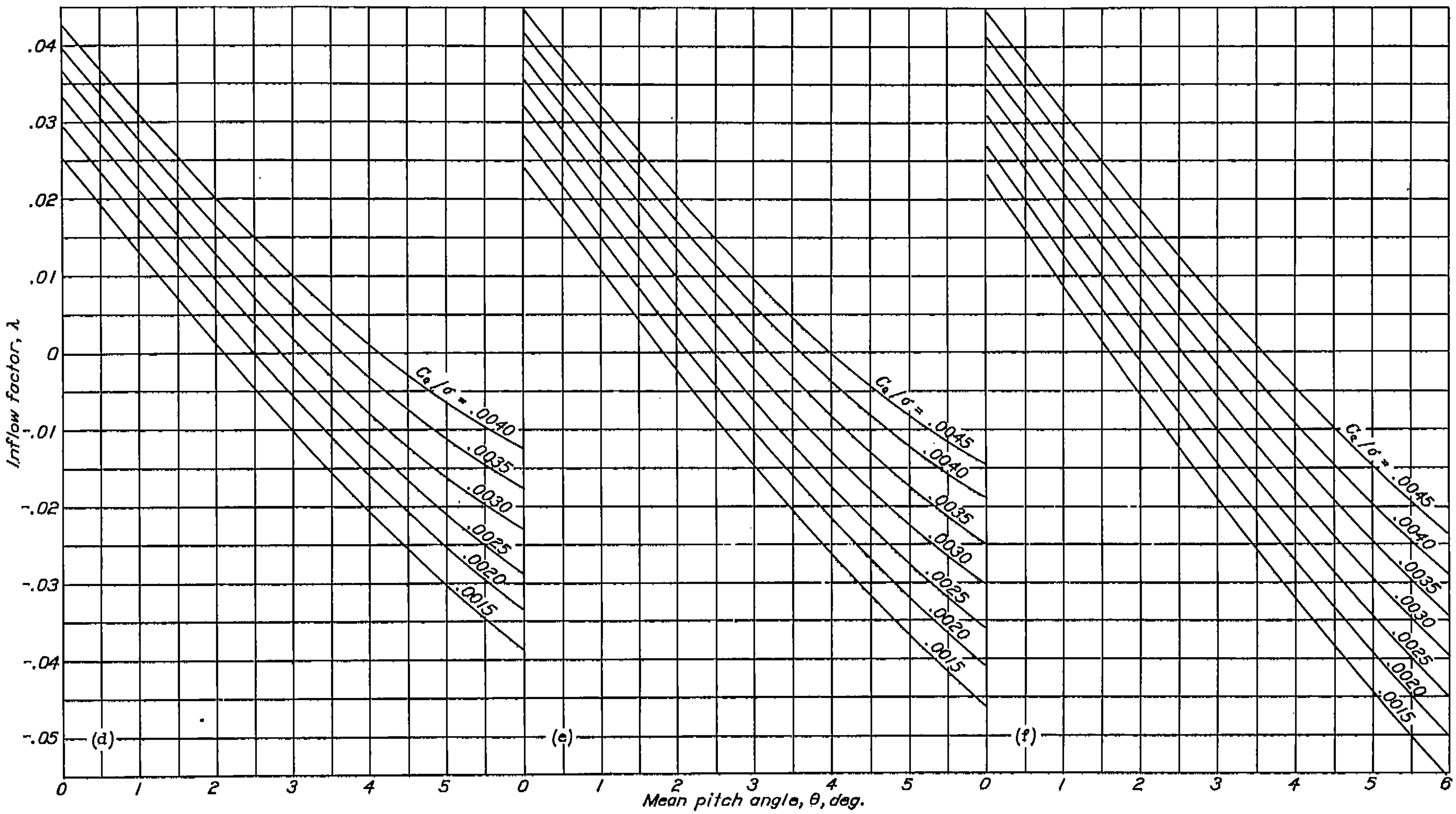
Expression (9) may then be reduced to

$$\frac{C_{Q_a}}{\sigma}=\frac{a}{2}\left\{K_1\lambda^2+[K_2\theta_0+K_3(\theta_1+\epsilon_0)+K_7\eta_1\right.$$
$$-(0.614\epsilon_2+0.141\eta_2)\mu^2]\lambda+K_4\theta_0^2$$
$$+K_5\theta_0(\theta_1+\epsilon_0)+K_6(\theta_1+\epsilon_0)^2+K_8\theta_0\eta_1$$
$$+K_9(\theta_1+\epsilon_0)\eta_1-[(0.284\epsilon_2+0.059\eta_2)\theta_0$$
$$\left.+(0.205\epsilon_2+0.047\eta_2)(\theta_1+\epsilon_0)-0.260\eta_1^2]\mu^2\right\} \tag{10}$$

Values of coefficients K_1 to K_9 may be found directly from figure 1 or for specified tip-speed ratios from table I.



(a) $\mu=0.30$ (b) $\mu=0.40$ (c) $\mu=0.45$



(d) μ=0.50

(e) μ=0.55

(f) μ=0.60

FIGURE 2.—Charts for determination of the inflow factor, λ.

Substitution of numerical values for the pitch setting and twist coefficients in equation (10) results in a quadratic in λ that may be solved for λ at any given value of C_{Q_a}/σ.

For rotors in which the pitch is constant along the radius of the blade and does not vary with azimuth position of the blade, equation (10) reduces to the form

$$\frac{C_{Q_a}}{\sigma}=\frac{a}{2}\{K_1\lambda^2+K_2\theta\lambda+K_4\theta^2\} \tag{11}$$

At tip-speed ratios above 0.4 with low pitch settings or above 0.3 with very high pitch settings, equations (9), (10), and (11) are inaccurate. The omission of terms of higher order than μ^4 becomes questionable and the overestimation of the lift coefficients of the stalled elements introduces a very serious error. Errors arising from the assumption that the angle ϕ is small are also too large to ignore. Consequently, some other method of integrating the basic expression must be found to extend the relation between torque and inflow to higher tip-speed ratios.

Reference to equation (2) shows that the indicated summation may also be made by evaluating the torque for a series of elements throughout the disk and integrating graphically over the disk area. This summation is accomplished for any combination of tip-speed ratio, pitch setting, and inflow velocity by first calculating u_P and u_T, the perpendicular and tangential components, respectively, of the velocity relative to each chosen element. The angle ϕ of the resultant velocity at each element to the plane of the disk is then $\tan^{-1} u_P/u_T$. Adding to ϕ the angle θ of the chord of the element to the plane of the disk, as defined by θ_0, θ_1, ϵ_0, ϵ_1, η_1, ϵ_2, and η_2, gives the angle of attack of each element. From a lift curve of the airfoil section, the corresponding lift coefficient of the element is determined. Where u_T is negative, the use of $-\phi-\theta$ in place of $\phi+\theta$ allows for the effect of the reversed flow over the blade in the same manner in which this problem was handled in the analytical expressions of references 1 and 2.

For rotors without blade twist, a sufficient number of graphical integrations of this type have been carried out to permit the construction of the charts of figure 2 from which the value of λ corresponding to a given C_{Q_a}/σ may be directly determined. Because accelerating and decelerating torques are equal in steady autorotation, the subscript has been omitted from the torque coefficient in figure 2. Comparison of the values of λ given by the charts with those obtained from the analytical expression (11) at the same C_{Q_a}/σ will demonstrate the error in expression (11). As in the analytical expression, values of $\gamma=15$ and $B=0.97$ have been used throughout the graphical integrations in the belief that the generality of the results is not appreciably impaired by these substitutions. A further limitation of the generality of the charts, imposed by the use throughout the graphical integrations of the lift characteristics of the N. A. C. A. 0012 airfoil section, is not considered serious because only small departures from these characteristics are to be expected in blade sections commonly used for rotors at the present time.

Obviously, it is impossible to supply charts covering all possible combinations of fixed and varying twist. It is reasonable to believe, however, that equations (10) and (11), although quantitatively in error as to the total accelerating torque produced by a given inflow velocity, correctly express the relative merit of rotors with and without twist as torque-generating devices. This belief amounts merely to the assumption that the percentage error inherent in the analytical method is the same for rotors with twist as for those without twist. Hence, it should be satisfactory at high tip-speed ratios to use equations (10) and (11) solely as a means of determining, for any rotor with twist, the pitch of what may be termed an "equivalent" constant-pitch rotor, that is, a rotor with no twist capable of generating the same accelerating torque at the same inflow velocity. The charts (fig. 2) for constant-pitch rotors may then be used to find the true value of the inflow velocity for the equivalent constant-pitch rotor, and the value so found may be considered to apply also to the original twisted rotor. In detail the procedure is as follows:

1. Substitute values for C_{Q_a}/σ, a, μ, K_1 to K_9, θ_0, θ_1, ϵ_0, η_1, ϵ_2, η_2, and solve equation (10) for λ.
2. Using this value of λ in equation (11), together with the original value of C_{Q_a}/σ, solve equation (11) for θ. The value of θ obtained is then the pitch of a constant-pitch rotor capable of generating the required torque at the same inflow velocity as the original rotor.
3. From the charts for constant-pitch rotors (fig. 2), determine the true inflow factor λ for a constant-pitch rotor of pitch θ generating torque C_{Q_a}/σ. The value of λ for the original rotor is, by hypothesis, identical with that of the equivalent constant-pitch rotor as found in step 3.

DECELERATING TORQUE

It will be noted that, in order to estimate the inflow velocity by the method of the preceding section, it is necessary to know the value of the accelerating torque C_{Q_a}/σ. On the strength of the physical requirement that accelerating and decelerating torques must be equal in steady autorotation, this quantity is normally determined by estimating the decelerating torque on the rotor from an integration over the disk area of the decelerating torque arising from the profile drag of a blade element. Hence

$$\frac{C_{Q_a}}{\sigma}=\frac{C_{Q_d}}{\sigma}=\frac{1}{2}\left(\frac{1}{2\pi}\int_0^{2\pi}d\psi\int_0^1 u_r^2C_D \sec \phi x dx\right) \tag{12}$$

The rigorous evaluation of the integral of expression (12) is possible only by graphical means because the elemental profile-drag coefficient C_D is a nonlinear function of both the angle of attack and the Reynolds Number at the element. Even then, uncertainty concerning the value of C_D in the presence of high radial velocities or in the stalled portion of the disk makes the validity of the results extremely doubtful. Nor does there appear to be any merit in the idea of replacing C_D sec ϕ by a mean value δ, as was done in reference 1, so that

$$\frac{1}{2\pi}\int_0^{2\pi}d\psi\int_0^1 u_T^2 C_D \sec\phi x dx=\delta\frac{1}{2\pi}\int_0^{2\pi}d\psi\int_0^1 u_T^2 x dx$$
$$=\delta\left(\frac{1}{4}+\frac{1}{4}\mu^2\right)$$

or, when the effect of the reversed-velocity region is included as in reference 1,

$$=\delta\left(\frac{1}{4}+\frac{1}{4}\mu^2-\frac{1}{32}\mu^4\right) \tag{13}$$

because both graphical integration and experiment indicate that the magnitude of δ will vary with both pitch

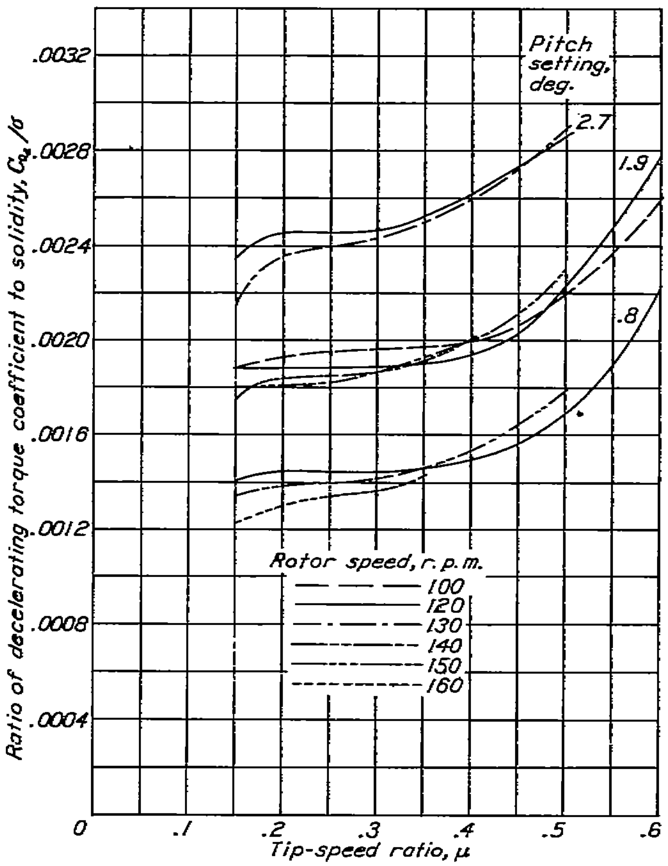


FIGURE 3.—Experimental decelerating torque of the PCA-2 rotor.

setting and tip-speed ratio and hence will require empirical information for its estimation. It seems fully as desirable and much simpler to formulate the empirical rules for the direct estimation of C_{Q_d}/σ. Unfortunately, the only reliable full-scale experimental information available for this purpose is confined to a single rotor, the PCA-2; and it is obviously impossible to establish such rules with any degree of finality at the present time. It does seem desirable, however, to study these data rather thoroughly in an effort to develop a tentative method of estimating C_{Q_d}/σ that may be expected to be subject to only minor alterations as additional data are accumulated.

Experimental data on C_{Q_d}/σ, from full-scale wind-tunnel tests of the PCA-2 rotor, are shown in figure 3. The values given were obtained by substituting experimental values of thrust and blade-motion coefficients in the thrust-coefficient expression of reference 2 and solving for λ. The value of C_{Q_a}/σ, and hence of C_{Q_d}/σ, corresponding to this value of λ was then obtained by a process the reverse of the one described in the preceding section of this paper.

A comparison of the values of C_{Q_d}/σ given in figure 3 with the experimental values of the ratio C_T/σ shown in figure 4 reveals that, at any given tip-speed ratio, C_{Q_d}/σ at all the pitch settings tested is almost directly proportional to C_T/σ. The closeness of this relationship is indicated in figure 5 where experimental values of the

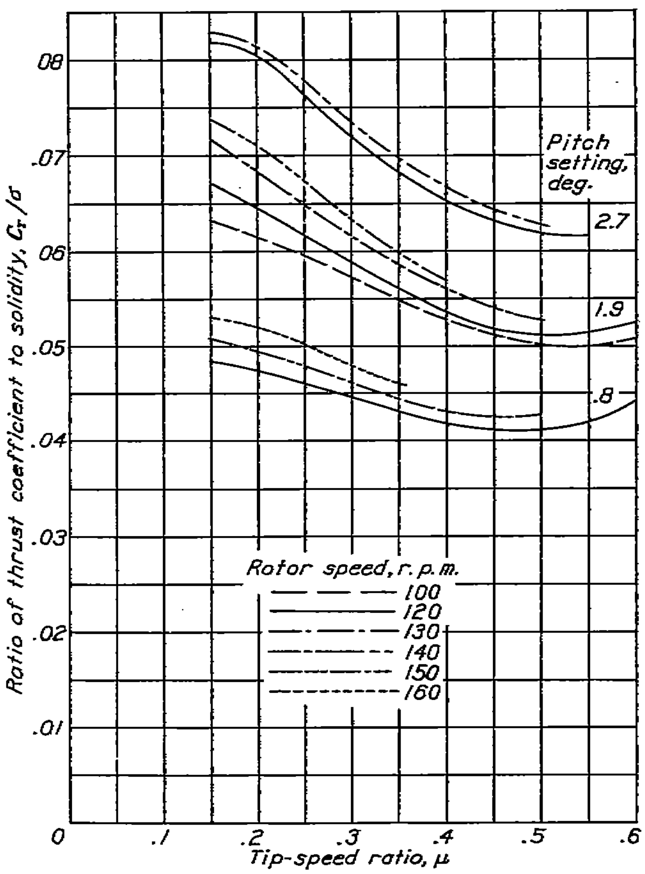


FIGURE 4.—Experimental thrust coefficient of the PCA-2 rotor.

quotient C_Q/C_T are plotted as functions of μ for the pitch settings given in figures 3 and 4.

From the designer's viewpoint, the curve shown in figure 5 provides a possible, although tedious, method of estimating the performance of a new design. As the various steps involved may not be instantly apparent, it seems desirable to outline briefly the recommended procedure.

Normally, from considerations of rotor efficiency, it is mandatory that the designer secure some definite value of C_T/σ at a particular tip-speed ratio. From this point, the problem becomes one of determining, first, the pitch angle required to satisfy this condition; and second, the performance of a rotor of this pitch as a function of the tip-speed ratio. On the assumption that the geometric and aerodynamic characteristics of the blades are known, the steps required in the solution are as follows:

1. Compute $(\theta_1+\epsilon_0)$, η_1, ϵ_2, and η_2 from the expressions of reference 3, neglecting the influence of the λ, θ_0, and $(\theta_1+\epsilon_0)$ terms.

2. From C_T/σ and μ find C_{Q_d}/σ from figure 5.

3. Assume a series of values of θ_0 and, using C_{Q_d}/σ from step 2, find the corresponding values of λ by the method described in the preceding section of this paper.

4. By substituting values of λ and θ_0 from step 3 into the thrust expression of reference 2, ignoring the terms involving a_1 and b_2, which almost cancel one another, obtain C_T/σ as a function of θ_0.

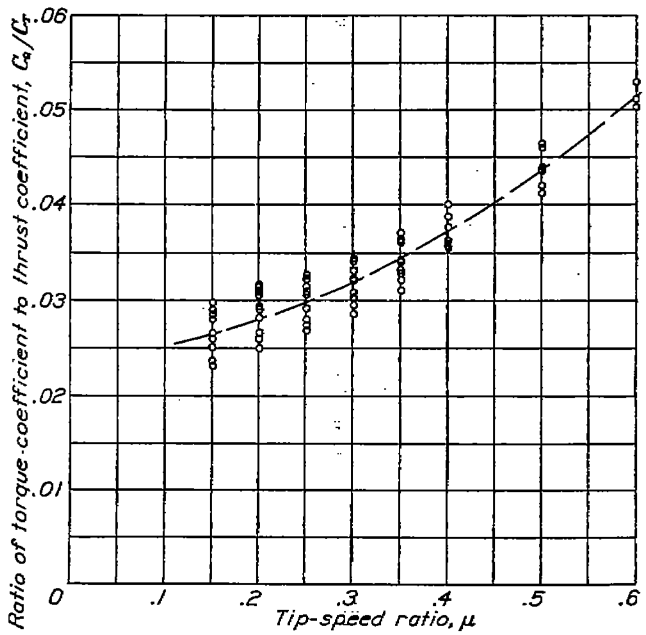


FIGURE 5.—Experimental torque-thrust ratio, C_Q/C_T, for the PCA-2 rotor.

5. Fix θ_0 at the value corresponding to the required C_T/σ.

The twist coefficients may now be recalculated without neglecting the λ, θ_0, and θ_1 terms and the entire process repeated to determine more accurately the required θ_0. The change in θ_0 resulting from this second approximation may be expected to be very small since the twist coefficients are only slightly affected by inflow velocity and pitch. The small change can probably be safely neglected except in unusual designs.

It will be noted, of course, that at low tip-speed ratios, where equations (10) and (11) are valid, λ may be obtained in step 3 as an analytical function of θ_0; this function may then be substituted into the expression for C_T/σ in step 4. The result will be a quadratic in θ_0 that can be solved directly for θ_0 at the design value of C_T/σ. Although this approach appears, at first glance, to be the more direct, in actual practice it will be found simpler to proceed as originally outlined, even at low tip-speed ratios.

The performance of the rotor of pitch θ_0 at other tip-speed ratios can be determined as follows:

1. Assume a value of C_T/σ at the new tip-speed ratio. Figure 4 may be used as a guide to a reasonable choice.

2. Obtain the corresponding C_{Q_d}/σ from figure 5.

3. Calculate ϵ_0, η_1, ϵ_2, and η_2 for the new tip-speed ratio by the method of reference 3, neglecting terms involving λ.

4. Determine λ by the method described in the first part of this paper.

5. Calculate C_T/σ from the thrust-coefficient expression given in reference 2 and check against the value assumed in step 1.

6. Repeat, modifying the assumed value until agreement is obtained.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *January 10, 1938.*

APPENDIX

Complete expressions follow for the coefficients K_1 to K_{31} appearing in equation (9). For convenience of reference, the term to which each coefficient is attached is given in parentheses immediately preceding the expression for the coefficient.

$$(\lambda^2),\ K_1=\frac{1}{2}B^2+\left(\frac{5}{4}+\frac{1}{1296}\gamma^2B^8\right)\mu^2+\left[\frac{1}{2B^2}+\frac{\gamma^2B^6}{(144+\gamma^2B^8)^2}\left(\frac{4}{3}-\frac{37}{162}\gamma^2B^8-\frac{77}{46656}\gamma^4B^{16}\right)\right]\mu^4$$

$$(\lambda\theta_0),\ K_2=\frac{1}{3}B^3+\left(\frac{8}{3}B+\frac{1}{864}\gamma^2B^9\right)\mu^2+\frac{2}{9\pi}\mu^3+\left[\frac{8}{3B}+\frac{\gamma^2B^7}{(144+\gamma^2B^8)^2}\left(\frac{224}{9}-\frac{11}{648}\gamma^2B^8-\frac{41}{31104}\gamma^4B^{16}\right)\right]\mu^4$$

$$(\lambda[\theta_1+\epsilon_0]),\ K_3=\frac{1}{4}B^4+\left(2B^2+\frac{1}{1080}\gamma^2B^{10}\right)\mu^2+\left[\frac{65}{32}+\frac{\gamma^2B^8}{(144+\gamma^2B^8)^2}\left(\frac{236}{15}-\frac{7}{108}\gamma^2B^8-\frac{47}{38880}\gamma^4B^{16}\right)\right]\mu^4$$

$$(\theta_0^2),\ K_4=\left(\frac{8}{9}B^2+\frac{1}{2304}\gamma^2B^{10}\right)\mu^2+\left[\frac{20}{9}+\frac{\gamma^2B^8}{(144+\gamma^2B^8)^2}\left(\frac{305}{36}+\frac{65}{1296}\gamma^2B^8-\frac{5}{82944}\gamma^4B^{16}\right)\right]\mu^4$$

$$(\theta_0[\theta_1+\epsilon_0]),\ K_5=\left(\frac{4}{3}B^3+\frac{1}{1440}\gamma^2B^{11}\right)\mu^2+\left[\frac{10}{3}B+\frac{\gamma^2B^9}{(144+\gamma^2B^8)^2}\left(\frac{57}{5}+\frac{7}{144}\gamma^2B^8-\frac{11}{51840}\gamma^4B^{16}\right)\right]\mu^4$$

$$([\theta_1+\epsilon_0]^2),\ K_6=\left(\frac{1}{2}B^4+\frac{1}{3600}\gamma^2B^{12}\right)\mu^2+\left[\frac{5}{4}B^2+\frac{\gamma^2B^{10}}{(144+\gamma^2B^8)^2}\left(\frac{92}{25}+\frac{1}{150}\gamma^2B^8-\frac{17}{129600}\gamma^4B^{16}\right)\right]\mu^4$$

$$(\lambda\eta_1),\ K_7=\frac{23}{30}B^3\mu+\left[\frac{11}{6}B+\frac{1}{864}\gamma^2B^9-\frac{1}{144+\gamma^2B^8}\,\frac{49}{2160}\gamma^2B^9+\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{32}{5}\gamma^2B^9+\frac{2}{45}\gamma^4B^{17}\right)\right]\mu^3$$

$$(\theta_0\eta_1),\ K_8=\frac{4}{15}B^4\mu+\left[\frac{89}{45}B^2+\frac{1}{1152}\gamma^2B^{10}-\frac{1}{144+\gamma^2B^8}\,\frac{53}{1728}\gamma^2B^{10}+\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{92}{15}\gamma^2B^{10}+\frac{23}{540}\gamma^4B^{18}\right)\right]\mu^3$$

$$([\theta_1+\epsilon_0]\eta_1),\ K_9=\frac{1}{5}B^5\mu+\left[\frac{89}{60}B^3+\frac{1}{1440}\gamma^2B^{11}-\frac{1}{144+\gamma^2B^8}\,\frac{43}{1800}\gamma^2B^{11}+\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{24}{5}\gamma^2B^{11}+\frac{1}{30}\gamma^4B^{19}\right)\right]\mu^3$$

$$(\lambda\epsilon_1),\ K_{10}=\frac{1}{720}\gamma B^7\mu+\left[-\frac{1}{540}\gamma B^5+\frac{1}{144+\gamma^2B^8}\left(\frac{1}{5}\gamma B^5+\frac{5}{1728}\gamma^3B^{13}\right)-\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{14}{45}\gamma^3B^{13}+\frac{7}{3240}\gamma^5B^{21}\right)\right]\mu^3$$

$$(\lambda\epsilon_2),\ K_{11}=\left[-\frac{3}{4}B^2-\frac{1}{144+\gamma^2B^8}\,\frac{16}{45}\gamma^2B^{10}+\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{384}{5}\gamma^2B^{10}+\frac{8}{15}\gamma^4B^{18}\right)\right]\mu^2$$

$$(\lambda\eta_2),\ K_{12}=\left[\frac{1}{72}\gamma B^6-\frac{1}{144+\gamma^2B^8}\left(\frac{24}{5}\gamma B^6+\frac{2}{45}\gamma^3B^{14}\right)+\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{56}{15}\gamma^3B^{14}+\frac{7}{270}\gamma^5B^{22}\right)\right]\mu^2$$

$$(\theta_0\epsilon_1),\ K_{13}=\frac{1}{960}\gamma B^8\mu+\left[-\frac{1}{2880}\gamma B^6+\frac{1}{144+\gamma^2B^8}\left(\frac{1}{72}\gamma B^6+\frac{5}{2304}\gamma^3B^{14}\right)-\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{7}{30}\gamma^3B^{14}+\frac{7}{4320}\gamma^5B^{22}\right)\right]\mu^3$$

$$(\theta_0\epsilon_2),\ K_{14}=\left[-\frac{1}{3}B^3-\frac{1}{144+\gamma^2B^8}\,\frac{17}{36}\gamma^2B^{11}+\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{368}{5}\gamma^2B^{11}+\frac{23}{45}\gamma^4B^{19}\right)\right]\mu^2$$

$$(\theta_0\eta_2),\ K_{15}=\left[\frac{1}{96}\gamma B^7-\frac{1}{144+\gamma^2B^8}\left(\frac{37}{15}\gamma B^7+\frac{1}{30}\gamma^3B^{15}\right)+\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{42}{15}\gamma^3B^{15}+\frac{7}{360}\gamma^5B^{23}\right)\right]\mu^2$$

$$([\theta_1+\epsilon_0]\epsilon_1),\ K_{16}=\frac{1}{1200}\gamma B^9\mu+\left[-\frac{1}{2400}\gamma B^7+\frac{1}{144+\gamma^2B^8}\left(\frac{1}{60}\gamma B^7+\frac{1}{576}\gamma^3B^{15}\right)-\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{14}{75}\gamma^3B^{15}+\frac{7}{5400}\gamma^5B^{23}\right)\right]\mu^3$$

$$([\theta_1+\epsilon_0]\epsilon_2),\ K_{17}=\left[-\frac{1}{4}B^4-\frac{1}{144+\gamma^2B^8}\,\frac{11}{30}\gamma^2B^{12}+\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{288}{5}\gamma^2B^{12}+\frac{2}{5}\gamma^4B^{20}\right)\right]\mu^2$$

$$([\theta_1+\epsilon_0]\eta_2),\ K_{18}=\left[\frac{1}{120}\gamma B^8-\frac{1}{144+\gamma^2B^8}\left(2\gamma B^8+\frac{2}{75}\gamma^3B^{16}\right)\right.$$
$$+\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{56}{25}\gamma^3B^{16}\right.$$
$$\left.\left.+\frac{7}{450}\gamma^5B^{24}\right)\right]\mu^2$$

$$(\epsilon_1^2),\ K_{19}=\left[-\frac{1}{144+\gamma^2B^8}\,\frac{1}{7200}\gamma^2B^{12}\right.$$
$$\left.+\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{2}{25}\gamma^2B^{12}+\frac{1}{1800}\gamma^4B^{20}\right)\right]\mu^2$$

$$(\epsilon_1\eta_1),\ K_{20}=\frac{1}{960}\gamma B^8\mu^2$$

$$(\eta_1^2),\ K_{21}=\left[\frac{22}{75}B^4-\frac{1}{144+\gamma^2B^8}\,\frac{1}{7200}\gamma^2B^{12}\right.$$
$$\left.+\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{2}{25}\gamma^2B^{12}+\frac{1}{1800}\gamma^4B^{20}\right)\right]\mu^2$$

$$(\epsilon_1\epsilon_2+\eta_1\eta_2),\ K_{22}=-\frac{1}{144+\gamma^2B^8}\,\frac{3}{50}\gamma B^9\mu$$

$$(\epsilon_1\eta_2-\eta_1\epsilon_2),\ K_{23}=\left[\frac{1}{144+\gamma^2B^8}\,\frac{1}{120}\gamma^2B^{13}\right.$$
$$\left.-\frac{1}{(144+\gamma^2B^8)^2}\left(\frac{48}{25}\gamma^2B^{13}+\frac{1}{75}\gamma^4B^{21}\right)\right]\mu$$

$$\left(\lambda\frac{M_W}{I_1\Omega^2}\right),\ K_{24}=-\frac{1}{108}\gamma B^6\mu^2$$
$$+\frac{1}{144+\gamma^2B^8}\left(\frac{47}{9}\gamma B^3+\frac{7}{486}\gamma^3B^{11}\right)\mu^4$$

$$\left(\theta_0\frac{M_W}{I_1\Omega^2}\right),\ K_{25}=-\frac{1}{144}\gamma B^6\mu^2$$
$$+\frac{1}{144+\gamma^2B^8}\left(\frac{485}{108}\gamma B^4+\frac{5}{1296}\gamma^3B^{12}\right)\mu^4$$

$$\left([\theta_1+\epsilon_0]\frac{M_W}{I_1\Omega^2}\right),\ K_{26}=-\frac{1}{180}\gamma B^7\mu^2$$
$$+\frac{1}{144+\gamma^2B^8}\left(\frac{18}{5}\gamma B^5+\frac{13}{3240}\gamma^3B^{13}\right)\mu^4$$

$$\left(\epsilon_1\frac{M_W}{I_1\Omega^2}\right),\ K_{27}=-\frac{1}{120}B^4\mu$$
$$+\left(\frac{1}{90}B^2-\frac{1}{144+\gamma^2B^8}\,\frac{17}{1080}\gamma^2B^{10}\right)\mu^3$$

$$\left(\eta_1\frac{M_W}{I_1\Omega^2}\right),\ K_{28}=\left(-\frac{1}{144}\gamma B^6+\frac{1}{144+\gamma^2B^8}\,\frac{17}{90}\gamma B^6\right)\mu^3$$

$$\left(\epsilon_2\frac{M_W}{I_1\Omega^2}\right),\ K_{29}=\frac{1}{144+\gamma^2B^8}\,\frac{34}{15}\gamma B^7\mu^2$$

$$\left(\eta_2\frac{M_W}{I_1\Omega^2}\right),\ K_{30}=\left(-\frac{1}{12}B^3+\frac{1}{144+\gamma^2B^8}\,\frac{17}{90}\gamma^2B^{11}\right)\mu^3$$

$$\left(\left[\frac{M_W}{I_1\Omega^2}\right]^2\right),\ K_{31}=\frac{1}{36}B^2\mu^2+\frac{7}{144}\mu^4$$

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TABLE I.—COEFFICIENTS K_1 TO K_9 OF ACCELERATING-TORQUE EXPRESSION FOR DIFFERENT TIP-SPEED RATIOS

Coefficient	Tip-speed ratio, μ									
	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
K_1	0.502	0.526	0.558	0.598	0.646	0.702	0.766	0.840	0.923	1.017
K_2	.369	.421	.490	.579	.689	.824	.980	1.179	1.407	1.675
K_3	.268	.306	.356	.420	.500	.598	.714	.854	1.018	1.210
K_4	.022	.040	.066	.100	.145	.203	.276	.368	.481	.618
K_5	.032	.058	.096	.146	.212	.296	.403	.536	.700	.900
K_6	.012	.021	.035	.053	.077	.108	.147	.195	.255	.328
K_7	.112	.156	.206	.264	.330	.407	.496	.598	.710	.850
K_8	.042	.063	.090	.125	.169	.223	.290	.369	.465	.576
K_9	.031	.046	.066	.091	.123	.163	.212	.270	.339	.421